

Hardening Blockchain Security with Formal Methods

FOR

5 Succinct

gnark-plonky2-verifier



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From Nov. 6, 2023 to Nov. 27, 2023, Succinct engaged Veridise to review the security of their gnark-plonky2-verifier project. The review covered the circuits implementing a plonky2 verifier. The circuits were implemented using the gnark zk-SNARK library*, which is written in the Go language. Veridise conducted the assessment over 12 person-weeks, with 4 engineers reviewing code over 3 weeks on commit 0x89b5a01e.

Closely following the circuit review, from Jan. 4 to Jan. 5, 2024, Succinct further engaged Veridise to review the security of their SuccinctGateway. This review covered two smart contracts written in Solidity inside of the succinctx repository[†]. The auditing strategy involved a tool-assisted analysis of the source code performed by Veridise engineers as well as extensive manual auditing.

Code assessment. The gnark-plonky2-verifier developers provided the source code of the gnark-plonky2-verifier circuits for review. To facilitate the Veridise auditors' understanding of the code, the gnark-plonky2-verifier developers provided a high-level overview of the implemented circuits as well as several references to material from the original plonky2 verifier[‡]. Following the audit, 2 minor changes to gnark-plonky2-verifier were also reviewed by Veridise, adding a small feature and slight optimizations.

Generally, the gnark-plonky2-verifier closely mirrored the original implementation of the plonky2 verifier so our auditing team could easily spot any deviations from the reference implementation. Except for some missing features, the Veridise auditors spotted very few omissions/deviations from the reference implementation. The source code also contained some documentation in the form of READMEs and documentation comments on core functions and structs.

The SuccinctGateway developers also provided the source code. These contracts are original to Succinct. The SuccinctGateway allows users to designate certain smart contracts as "verifier contracts," then request whitelisted sets of provers to perform on-chain calls with inputs which satisfy the verifiers. The developers worked with Veridise to provide additional information about the behavior and intended usage, answering several questions about the protocol.

The source code contained a test suite. The tests across the entire suite covered, on average, 79.1% of the gnark-plonky2-verifier program statements under test (with a standard deviation of 22.5), and 71.2% of the contract statements.

Summary of issues detected. The audit uncovered 39 issues, 3 of which are assessed to be of high or critical severity by the Veridise auditors. All three of these issues (V-SCT-VUL-001,

^{*} https://github.com/succinctlabs/gnark-plonky2-verifier/tree/89b5a01e4b

[†] https://github.com/succinctlabs/succinctx/tree/abd43565

[‡]https://github.com/0xPolygonZero/plonky2/tree/64cc1000e7

V-SCT-VUL-002, and V-SCT-VUL-003) were caused by missing constraints on values provided by hints[§]. No issues of high or critical severity were identified inside the smart contracts.

The Veridise auditors also identified several medium-severity issues, including a missing range check in the evaluation proofs (V-SCT-VUL-005), the improper use of a function which may mutate its arguments (V-SCT-VUL-004), and the possibility of locked funds in the contract (V-SCT-VUL-005). Finally, the audit uncovered a number of minor issues, including slightly over-constrained circuits (V-SCT-VUL-007, V-SCT-VUL-012), ignored parameters (V-SCT-VUL-033), minor logical errors (V-SCT-VUL-009, V-SCT-VUL-010, V-SCT-VUL-028) and various maintainability issues.

As of commit 0xc01f530fe, Succinct has fixed all of the identified issues in the gnark-plonky2-verifier. The SuccinctGateway developers have also provided fixes to all of the identified issues. Only the warning issue V-SCT-VUL-026 remains partially unresolved. However, any centralization risk to the users has been mitigated by allowing custom prover whitelists. The only remaining risk is left to the contract operators, as the fee vault may be changed if the owner is compromised, or users who choose to rely on the default prover list.

Recommendations. After auditing the protocol, the auditors had a few suggestions to improve the gnark-plonky2-verifier.

Extended type system for field emulation. Succinct's type system does much to improve the readability and safety of their circuits. One key distinction which is not included in the type system, but could greatly simplify reasoning about the code, is distinguishing between reduced Goldilocks Field elements and unreduced field elements. This change would have two positive impacts. First, by limiting the interface for constructing a reduced Goldilocks field element, one may know whether the underlying representation of the Goldilocks field element is reduced from the type alone. This drastically reduces the number of caller assumptions in the codebase. Second, by keeping track programmatically of the maximum number of bits required to represent a given unreduced Goldilocks field elements, reductions may be performed "just-in-time" rather than via manual reasoning.

More detailed reference comments. Since Succinct's code base is almost a direct port of Polygon Zero's plonky2 repository, most of the functions and arguments match the reference implementation almost exactly. However, certain parts of the code are altered or moved across the codebase for efficiency (see, for example, V-SCT-VUL-018). This can lead to some confusion for readers, and may be difficult to keep in sync with changes to the plonky2 repository. We would recommend documenting each deviation from the reference implementation, preferably with a link to the relevant source code. If possible, providing a link to the reference implementation of each function would greatly improve readability, as the plonky2 code repository contains relevant comments and documentation.

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[§]https://docs.gnark.consensys.net/HowTo/write/hints

Table 2.1: Application Summary.

Name	Version	Type	Platform
gnark-plonky2-verifier	0x89b5a01e	Go	gnark
SuccinctGateway	0xb1f6bd08 - 0xabd43565	Solidity	Ethereum

Table 2.2: Engagement Summary.

Dates	Method	Consultants Engaged	Level of Effort
Nov. 6 - Nov. 27, 2023	Manual & Tools	4	12 person-weeks
Jan. 4 - Jan. 5, 2024	Manual & Tools	2	4 person-days

Table 2.3: Vulnerability Summary.

Name	Number	Resolved
Critical-Severity Issues	3	3
High-Severity Issues	0	0
Medium-Severity Issues	3	3
Low-Severity Issues	9	9
Warning-Severity Issues	12	11
Informational-Severity Issues	12	12
TOTAL	39	38

Table 2.4: Category Breakdown.

Name	Number
Maintainability	13
Logic Error	11
Data Validation	5
Underconstrained Circuit	3
Overconstrained Circuit	2
Locked Funds	1
Denial of Service	1
Access Control	1
Constraint Optimization	1
Gas Optimization	1

3.1 Audit Goals

The engagement was scoped to provide a security assessment of gnark-plonky2-verifier's zero-knowledge circuits and the as outlined below. In our audit, we sought to answer the following questions:

- ▶ Are any circuit wires under-constrained?
- ► Can a generated witness deviate from expected behavior?
- ▶ Do all parameters have bound checks so that computations do not over/underflow?
- ▶ Are there unnecessary constraints that only increase verifier overhead?
- ▶ Are Goldilocks Field computations properly emulated within the BN254 field?
- ▶ Do the PLONK gates contain arithmetic errors?
- ▶ Will the implementation of Fast Reed-Solomon IOPPs accept invalid commitments?
- ▶ Does the verifier accept invalid proofs?
- ► Can any non-permissioned entity execute an unexpected method in the gateway?
- ► Can the calls and callbacks requested by users be exploited via careful choice of the callback address?
- ► Are any common Solidity vulnerabilities (reentrancy, large stake holder attacks, unchecked return values, etc.) present in the contracts?
- ▶ Can funds be locked in the contracts?

3.2 Audit Methodology & Scope

Audit Methodology. To address the questions above, our audit involved a combination of human experts and automated program analysis & testing tools. In particular, we conducted our audit with the aid of the following techniques:

- ► *Static analysis*. To identify potential common vulnerabilities, we leveraged our custom smart contract analysis tool Vanguard. This tools are designed to find instances of common smart contract vulnerabilities, such as reentrancy and uninitialized variables.
- ► Fuzzing/Property-based Testing. We leverage fuzz testing to determine if the protocol may deviate from the expected behavior (see Chapter 5 for discussion and results). To do this, we formalize the desired behavior of the protocol and then perform differential fuzzing to determine if a violation of the specification can be found.
- ▶ Formal Verification. We also leverage our custom formal verification tool Picus to verify safety properties of the zero-knowledge circuits (see Chapter 6 for discussion and results). This tool is designed to prove or find violations of determinism, which is an important safety property for zero-knowledge circuits.

Scope. The scope of this audit is limited to the Go language files in the following directories of the repository located at https://github.com/succinctlabs/gnark-plonky2-verifier (excluding all *_utils.go, *_test.go, *_constants.go, util.go, and vars.go files):

- ▶ challenger/
- ▶ fri/
- ▶ goldilocks/
- plonk/ and plonk/gates/
- ▶ poseidon/
- ▶ verifier/

For the smart contracts, only the SuccinctGateway.sol and FunctionRegistry.sol files were in scope.

During the audit, the Veridise auditors referred to the excluded files but assumed that they have been implemented correctly.

Methodology. Veridise auditors inspected the provided tests and read the documentation for both gnark-plonky2-verifier and the SuccinctGateway. They then began a manual audit of the code assisted by both automated testing and verification tools. During the audit, the Veridise auditors regularly met with the gnark-plonky2-verifier developers to ask questions about the code. The Veridise auditors also referred to the original Plonky2 implementation (occasionally cited as "the reference implementation") during the course of the audit to check for differences in gnark-plonky2-verifier's implementation.

3.3 Classification of Vulnerabilities

When Veridise auditors discover a possible security vulnerability, they must estimate its severity by weighing its potential impact against the likelihood that a problem will arise. Table 3.1 shows how our auditors weigh this information to estimate the severity of a given issue.

Table 3.1: Severity Breakdown.

	Somewhat Bad	Bad	Very Bad	Protocol Breaking
Not Likely	Info	Warning	Low	Medium
Likely	Warning	Low	Medium	High
Very Likely	Low	Medium	High	Critical

In this case, we judge the likelihood of a vulnerability as follows in Table 3.2:

Table 3.2: Likelihood Breakdown

Not Likely	A small set of users must make a specific mistake
	Requires a complex series of steps by almost any user(s)
Likely	- OR -
	Requires a small set of users to perform an action
Very Likely	Can be easily performed by almost anyone

In addition, we judge the impact of a vulnerability as follows in Table 3.3:

Table 3.3: Impact Breakdown

Somewhat Bad	Inconveniences a small number of users and can be fixed by the user
	Affects a large number of people and can be fixed by the user
Bad	- OR -
	Affects a very small number of people and requires aid to fix
	Affects a large number of people and requires aid to fix
Very Bad	- OR -
	Disrupts the intended behavior of the protocol for a small group of
	users through no fault of their own
Protocol Breaking	Disrupts the intended behavior of the protocol for a large group of
_	users through no fault of their own
	-

In this section, we describe the vulnerabilities found during our audit. For each issue found, we log the type of the issue, its severity, location in the code base, and its current status (i.e., acknowledged, fixed, etc.). Table 4.1 summarizes the issues discovered:

Table 4.1: Summary of Discovered Vulnerabilities.

ID	Description	Severity	Status
V-SCT-VUL-001	inverse missing range check	Critical	Fixed
V-SCT-VUL-002	Unconstrained variable in Reduce()	Critical	Fixed
V-SCT-VUL-003	Unconstrained variable in ReduceWithMaxBits()	Critical	Fixed
V-SCT-VUL-004	MulAdd() may mutate arguments	Medium	Fixed
V-SCT-VUL-005	Missing range-check on EvalProofs	Medium	Fixed
V-SCT-VUL-006	Contracts: funds may be locked when feeVault is	Medium	Fixed
V-SCT-VUL-007	FRI Parameter Ignored During Loading	Low	Fixed
V-SCT-VUL-008	interpolate() over-constrained	Low	Fixed
V-SCT-VUL-009	ArityBits = 8 will provoke errors	Low	Fixed
V-SCT-VUL-010	alpha value overwritten to 1	Low	Fixed
V-SCT-VUL-011	Bitpacking operation may lead to overlap	Low	Fixed
V-SCT-VUL-012	evalL0 overconstrained	Low	Fixed
V-SCT-VUL-013	Salted evaluation not considered	Low	Fixed
V-SCT-VUL-014	Index calculation discrepancy	Low	Fixed
V-SCT-VUL-015	Contracts: _callbackGasLimit is unused	Low	Fixed
V-SCT-VUL-016	1 added to TwoAdicSubgroup output twice	Warning	Fixed
V-SCT-VUL-017	Missed opportunities to use abstractions	Warning	Fixed
V-SCT-VUL-018	Unused functions/variables	Warning	Fixed
V-SCT-VUL-019	Bit reversal incorrect for ArityBits != 4	Warning	Fixed
V-SCT-VUL-020	clearBuffer doesn't clear the buffer	Warning	Fixed
V-SCT-VUL-021	Possible leakage of randomness	Warning	Fixed
V-SCT-VUL-022	Unused challenger parameter due to relocated logic	Warning	Fixed
V-SCT-VUL-023	Errors in gate Id functions	Warning	Fixed
V-SCT-VUL-024	Use of incorrect constant	Warning	Fixed
V-SCT-VUL-025	Missing degree check for CosetInterpolationGate	Warning	Fixed
V-SCT-VUL-026	Contracts: Centralization Risk	Warning P	artially Fixed
V-SCT-VUL-027	Contracts: All _callbackAddresses permitted	Warning	Fixed
V-SCT-VUL-028	Replace GoldilocksHashOut size with named constant	Info	Fixed
V-SCT-VUL-029	Type error in bn254's HashOrNoop()	Info	Fixed
V-SCT-VUL-030	Non-std range-check used	Info	Fixed
V-SCT-VUL-031	Various out-of-date comments and documentation	Info	Fixed
V-SCT-VUL-032	Typos in code	Info	Fixed
V-SCT-VUL-033	Ignored gate parameters	Info	Fixed
V-SCT-VUL-034	Hard-coded constants in code	Info	Fixed
V-SCT-VUL-035	No assertion that gnark is using BN254	Info	Fixed
V-SCT-VUL-036	Inaccurate bounds-check on numConsts	Info	Fixed
V-SCT-VUL-037	Sub-optimal sub-expression in random access gate	Info	Fixed
V-SCT-VUL-038	Contracts: Code Recommendations	Info	Fixed
V-SCT-VUL-039	Contracts: Possible Wasted Gas	Info	Fixed

4.1 Detailed Description of Issues

4.1.1 V-SCT-VUL-001: inverse missing range check

Severity	Critical	Commit	89b5a01
Type	Underconstrained Circuit	Status	Fixed
File(s)	goldilocks/base.go		
Location(s)	Inverse()		
Confirmed Fix At	9e96393		

The goldilocks/base.go file implements standard operations for the Goldilocks field emulated within the base field. One of these operations is Inverse(), which computes the inverse of a Goldilocks field element.

The actual computation of the inverse is done using a hint. To ensure correctness, the inverse of x is constrained to multiply with x to equal 1. This can be seen in the implementation below:

```
func (p *Chip) Inverse(x Variable) Variable {
    result, err := p.api.Compiler().NewHint(InverseHint, 1, x.Limb)
    if err != nil {
        panic(err)
    }

    inverse := NewVariable(result[0])
    product := p.Mul(inverse, x)
    p.api.AssertIsEqual(product.Limb, frontend.Variable(1))
    return inverse
]
```

Snippet 4.1: Implementation of Inverse()

However, in the above code, p.Mul assumes that inverse and x are reduced elements of the Goldilocks field. More specifically, p.Mul(inverse, x) asserts that there exist field elements quotient and remainder such that

```
inverse * x == quotient * MODULUS + remainder
d <= quotient < MODULUS
d <= remainder < MODULUS</pre>
```

Since the remainder is returned as the value of the product, the constraints that Inverse() encodes can be summarized as follows:

```
inverse * x == quotient * MODULUS + remainder
do <= quotient < MODULUS
do <= remainder < MODULUS
remainder == 1</pre>
```

Writing inv(x) for the inverse of x within the base field, we can solve for inverse.

```
1 inverse := inv(x) * (quotient * MODULUS + 1)
```

Since inverse is not constrained to be in the Goldilocks field, this is a valid solution for any θ <= quotient < MODULUS. Only one of these MODULUS-many solutions is the correct reduced inverse.

Impact Inverse() may return an element that is not a member of the Goldilocks field and is not equal to the Goldilocks-inverse of x when reduced into the field.

To see that the latter statement holds, take any true Goldilocks-inverse xInv of x with non-zero quotient, and note that xInv - MODULUS is a valid solution to the Inverse() constraints.

Recommendation Perform a range check on inverse, requiring it to be a member of the Goldilocks field.

4.1.2 V-SCT-VUL-002: Unconstrained variable in Reduce()

Severity	Critical	Commit	89b5a01
Type	Underconstrained Circuit	Status	Fixed
File(s)	goldilocks/base.go		
Location(s)	Reduce()		
Confirmed Fix At	297a820		

The <code>goldilocks/base.go</code> file emulates the Goldilocks field within the base field. One important operation is <code>Reduce()</code>, which maps an element of the base field down to an equivalent (modulo the order of the Goldilocks field) element of the Goldilocks field.

The actual computation of the reduction is done using a hint. However, the result of the hint is never included in a constraint with the variable x which is intended to be reduced.

```
func (p *Chip) Reduce(x Variable) Variable {
1
      // Witness a 'quotient' and 'remainder' such that:
2
3
      //
4
      //
              MODULUS * quotient + remainder = x
      //
      // Must check that offset \in [0, MODULUS) and carry \in [0, 2^
6
      RANGE_CHECK_NB_BITS) to ensure
      // that this computation does not overflow. We use 2^RANGE_CHECK_NB_BITS to
      reduce the cost of the range check
8
      // In other words, we assume that we at most compute a a dot product with
9
      dimension at most RANGE_CHECK_NB_BITS - 128.
10
      result, err := p.api.Compiler().NewHint(ReduceHint, 2, x.Limb)
11
       if err != nil {
12
          panic(err)
13
14
15
16
      quotient := result[0]
       p.rangeChecker.Check(quotient, RANGE_CHECK_NB_BITS)
17
18
       remainder := NewVariable(result[1])
19
20
       p.RangeCheck(remainder)
21
       return remainder
22 }
```

Snippet 4.2: Implementation of Reduce()

Since result is computed from x by a hint, that computation creates no constraints. As a result, any quotient, remainder pair which satisfy the range checks will satisfy the entire Reduce() constraints.

Impact When Reduce()ing to the Goldilocks field, an attacker may choose any Goldilocks field element they desire.

Recommendation Require MODULUS * quotient + remainder to equal x.

4.1.3 V-SCT-VUL-003: Unconstrained variable in ReduceWithMaxBits()

Severity	Critical	Commit	89b5a01
Type	Underconstrained Circuit	Status	Fixed
File(s)	goldilocks/base.go		
Location(s)	ReduceWithMaxBits()		
Confirmed Fix At	297a820		

The function ReduceWithMaxBits() performs a reduction on a variable x which is assumed to be representable in maxNbBits. The implementation suffers from the same lack of constraints described in V-SCT-VUL-002: x is not constrained by ReduceWithMaxBits().

```
func (p *Chip) ReduceWithMaxBits(x Variable, maxNbBits uint64) Variable {
       // Witness a 'quotient' and 'remainder' such that:
2
       //
3
       //
               MODULUS * quotient + remainder = x
4
5
       //
       // Must check that remainder \in [0, MODULUS) and quotient \in [0, 2^maxNbBits)
6
       to ensure that this
       // computation does not overflow.
8
       result, err := p.api.Compiler().NewHint(ReduceHint, 2, x.Limb)
       if err != nil {
10
11
           panic(err)
12
       }
13
       quotient := result[0]
14
       p.rangeChecker.Check(quotient, int(maxNbBits))
15
16
       remainder := NewVariable(result[1])
17
       p.RangeCheck(remainder)
18
       return remainder
19
20 }
```

Snippet 4.3: Implementation of ReduceWithMaxBits()

Since result is computed from x by a hint, that computation creates no constraints. As a result, any quotient, remainder pair which satisfy the range checks will satisfy the entire ReduceWithMaxBits() constraints.

Impact When calling ReduceWithMaxBits() to map an element into the Goldilocks field, an attacker may choose any Goldilocks field element they desire.

Recommendation Require MODULUS * quotient + remainder to equal x.

4.1.4 V-SCT-VUL-004: MulAdd() may mutate arguments

Severity	Medium		Commit	89b5a01
Type	Logic Error		Status	Fixed
File(s)		goldilo	cks/base.go	
Location(s)	MulAddNoReduce(), MulAdd()			
Confirmed Fix At		a	6707ed	

The documentation for frontend.API.MulAcc states that it *may* modify the first argument:

```
1  // MulAcc sets and return a = a + (b*c).
2  //
3  // ! The method may mutate a without allocating a new result. If the input
4  // is used elsewhere, then first initialize new variable, for example by
5  // doing:
6  //
7  // acopy := api.Mul(a, 1)
8  // acopy = MulAcc(acopy, b, c)
9  //
10  // ! But it may not modify a, always use MulAcc(...) result for correctness.
11 MulAcc(a, b, c Variable) Variable
```

Snippet 4.4: Documentation of frontend.API.MulAcc

Both MulAdd and MulAccNoReduce in goldilocks/base.go call MulAcc directly on c.Limb (from argument c). For example, see the below definition of MulAddNoReduce().

```
func (p *Chip) MulAddNoReduce(a Variable, b Variable, c Variable) Variable {
    return NewVariable(p.api.MulAcc(c.Limb, a.Limb, b.Limb))
}
```

Snippet 4.5: Definition of MulAddNoReduce()

Impact If the c.Limb frontend.Variable is a pointer type (which is allowed and does occur in this codebase), it is possible for MulAdd and MulAddNoReduce to mutate their arguments and propagate unintended modifications to other callsites. This potentially affects many locations, since MulAdd and MulAddNoReduce are the base operations of many other functions, including Add and Sub in goldilocks/base.go, AddExtension and SubExtension in goldilocks/quadratic_extension.go, and AddExtensionAlgebra and SubExtensionAlgebra in goldilocks/quadratic_extension_algebra.go. Some possible affected callsites where MulAcc may affect subsequent computation include:

- evalVanishingPoly in plonk/plonk.go: wireValuePlusGamma may be modified and taint the denominator calculation.
- ► EvalUnfiltered in plonk/gates/poseidon_gate.go: deltaI may be modified and taint the state[i+4] calculation.
- ▶ friCombineInitial in fri/fri.go: subgroupX_QE may be modified.
- ▶ interpolate in fri/fri.go: x may be modified for future loop iterations.
- EvalUnfiltered in plonk/gates/constant_gate.go: localConstants may be modified.
- ► EvalUnfiltered in plonk/gates/public_input_gate.go: localWires may be modified.

Recommendation Follow the recommendation in the gnark documentation and copy c.Limb before performing the MulAcc operation for safety:

```
1 cLimbCopy := p.api.Mul(c.Limb, 1)
2 p.api.MulAcc(cLimbCopy, a.Limb, b.Limb)
```

Developer Response We now make a copy of the first argument before passing it to frontend. api.MulAcc in all cases where this could be an issue.

4.1.5 V-SCT-VUL-005: Missing range-check on EvalProofs

Severity	Medium		Commit	89b5a01
Type	Data Validation		Status	Fixed
File(s)	verifier/verifier.go			
Location(s)	rangeCheckProof()			
Confirmed Fix At			6af5b0a	

Only the first item of the InitialTreesProof.EvalProofs list is range checked (i.e., only on queryRound.InitialTreesProof.EvalsProofs[0]):

```
// Range check the openings proof.
   for _, queryRound := range proof.OpeningProof.QueryRoundProofs {
3
         for _, initialTreesElement := range queryRound.InitialTreesProof.EvalsProofs
       [0].Elements {
           c.glChip.RangeCheck(initialTreesElement)
4
6
7
         for _, queryStep := range queryRound.Steps {
8
9
             for _, eval := range queryStep.Evals {
10
                     c.glChip.RangeCheckQE(eval)
11
             }
12
         }
13 }
```

Snippet 4.6: Range check performed on the openings proof.

Impact The values of EvalsProofs are used in fri/fri.go (in verifyInitialProof and in friCombineInitial), so the non-range-checked inputs could result in the violation of other assumptions in the codebase.

For example, the unchecked Elements are eventually hashed in BN254Chip. HashNoPad (verifyInitialProof \rightarrow verifyMerkleProofToCapWithCapIndex \rightarrow HashOrNoop \rightarrow HashNoPad), which assumes that the inputs are reduced goldilocks field elements (as the bit-packing operation assumes the inputs are at most 64 bits).

Recommendation Perform range checks on all elements of the EvalsProofs list.

4.1.6 V-SCT-VUL-006: Contracts: funds may be locked when feeVault is disabled

Severity	Medium	Commit	abd4356
Type	Locked Funds	Status	Fixed
File(s)	contracts/src/SuccinctGateway.sol		
Location(s)	SuccinctGateway		
Confirmed Fix At	https://github.com/succinctlabs/succinctx/pull/312		

The SuccinctGateway contract allows a privileged user (called the guardian) to set the address of the IFeeVault contract that will hold fees paid by users for their requests. This is done via the setFeeVault() function.

```
1 /// @dev Sets the fee vault to a new address. Can be set to address(0) to disable
    fees.
2 /// @param _feeVault The address of the fee vault.
3 function setFeeVault(address _feeVault) external onlyGuardian {
    emit SetFeeVault(feeVault, _feeVault);
    feeVault = _feeVault;
6 }
```

Snippet 4.7: Definition of SuccinctGateway.setFeeVault()

The documentation states that the fee vault address "can be set to address(0) to disable fees" and both the requestCall() and requestCallback() functions correctly ensure feeVault != address (0) before depositing the msg.value into the feeVault contract. However, when feeVault == address(0) the msg.value funds sent by msg.sender with the transaction are retained in the SuccinctGateway contract with no way for the user to retrieve them.

Impact If feeVault == address(0) and a user sends funds when calling the requestCall() or requestCallback() functions, those funds will be locked in the SuccinctGateway contract.

Recommendation The SuccinctGateway contract should provide a way for the user to retrieve their funds that were locked because feeVault == address(0).

Developer Response We added a recover() function to the SuccinctGateway to allow the guardian to initiate a fund transfer to a given address.

4.1.7 V-SCT-VUL-007: FRI Parameter Ignored During Loading

Severity	Low		Commit	89b5a01
Type	Logic Error		Status	Fixed
File(s)	types/common_data.go			
Location(s)	ReadCommonCircuitData()			
Confirmed Fix At			f71795a	

Note: This is technically out of scope, but our team noticed this issue while reviewing the type representation of the different elements inside the codebase.

Function ReadCommonCircuitData populates all fields for an instance of type CommonCircuitData, which is then returned to the caller. However, when populating the FriParams (see snippet bellow), parameter Hiding is ignored.

```
commonCircuitData.FriParams.Config.RateBits = raw.FriParams.Config.RateBits
commonCircuitData.FriParams.Config.CapHeight = raw.FriParams.Config.CapHeight
commonCircuitData.FriParams.Config.ProofOfWorkBits = raw.FriParams.Config.
    ProofOfWorkBits
commonCircuitData.FriParams.Config.NumQueryRounds = raw.FriParams.Config.
    NumQueryRounds
commonCircuitData.FriParams.ReductionArityBits = raw.FriParams.ReductionArityBits
```

Snippet 4.8: Snippet from ReadCommonCircuitData

Impact Function validateFriProofShape might panic for proofs whose Hiding parameter is set to true. This can negatively impact some users when trying to generate a proof.

Recommend loading the Hiding parameter in function Read Common Circuit Data

Developer Response We have decided to require the Hiding parameter to be set to false.

4.1.8 V-SCT-VUL-008: interpolate() over-constrained

Severity	Low	Commit	89b5a01
Type	Overconstrained Circuit	Status	Fixed
File(s)	fri/fri.go		
Location(s)	interpolate()		
Confirmed Fix At	d241f54		

The interpolate() function takes a vector of xPoints and yPoints as inputs and uses them to evaluate the polynomial they define at a new point x. This is handled in two cases:

- 1. When x is not equal to any xPoints[i].
- 2. When x is equal to some xPoints[i].

In case 1, a division with a denominator of x - xPoints[i] is computed.

Snippet 4.9: Division by x - xPoints[i] in interpolate().

Case 2, later on in the function, uses selectors to evaluate to yPoints[i] if x = xPoints[i].

However, DivExtension computes the Inverse directly, requiring it to exist.

```
func (p *Chip) InverseExtension(a QuadraticExtensionVariable)
    QuadraticExtensionVariable {
    a0IsZero := p.api.IsZero(a[0].Limb)
    a1IsZero := p.api.IsZero(a[1].Limb)
    p.api.AssertIsEqual(p.api.Mul(a0IsZero, a1IsZero), frontend.Variable(0))
```

Snippet 4.10: The beginning of InverseExtension(), which asserts the existence of an inverse.

So, whenever case 2 occurs, the constraints for case 1 will be unsatisfiable.

Impact Evaluating the polynomial at one of the xPoints using interpolate() results in unsatisfiable constraints.

Recommendation To make case 2 satisfiable, the developers need to perform an unsafe division, and then use the yPoints whenever the denominator is zero.

See also V-SCT-VUL-012.

Developer Response [If applicable]

4.1.9 V-SCT-VUL-009: ArityBits = 8 will provoke errors

Severity	Low	Commit	89b5a01
Type	Logic Error	Status	Fixed
File(s)	fri/fri.go		
Location(s)	computeEvaluation()		
Confirmed Fix At		c0cbac8	

The computeEvaluation() function uses an arityBits of at most 8 to evaluate the next reduced FRI polynomial. evals, which is used to perform this computation, is an array of length arity := 2^arityBits.

```
func (f *Chip) computeEvaluation(
    // [VERIDISE] ...

gl.QuadraticExtensionVariable {
    arity := 1 << arityBits
    if (len(evals)) != arity {
        panic("len(evals) ! arity")
    }

if arityBits > 8 {
    panic("currently assuming that arityBits is <= 8")
}</pre>
```

Snippet 4.11: Checks at the beginning of computeEvaluation()

Later, when permuting the evaluations, 8-bit indices are used.

```
permutedEvals := make([]gl.QuadraticExtensionVariable, len(evals))
for i := uint8(0); i < uint8(len(evals)); i++ {
    newIndex := bits.Reverse8(i) >> arityBits
    permutedEvals[newIndex] = evals[i]
}
```

Snippet 4.12: Evaluation of permutedEvals.

Casting len(evals) to uint8 will truncate any values larger than 255. If arityBits == 8, then len(evals) == 256, and uint8(len(evals)) evaluates to zero.

Impact For an arityBits of 8, this function will cause errors due to invalid memory accesses. Currently, the only callers use arityBits == 4, so this will only be an issue in future iterations of the protocol.

Recommendation Require arityBits to be at most 7, or handle the arityBits == 8 case separately.

Developer Response We now use a <= bounds-check with uint8(len(evals)-1).

4.1.10 V-SCT-VUL-010: alpha value overwritten to 1

Severity	Low	Commit	89b5a01
Type	Logic Error	Status	Fixed
File(s)	poseidon/bn254.go		
Location(s)	HashOrNoop()		
Confirmed Fix At		5766879	

The big.Int.Exp function documentation states that, for z.Exp(x, y, m):

Exp sets $z = x^{**}y \mod |m|$ (i.e. the sign of m is ignored), and returns z.

So, in the loop body:

Snippet 4.13: Loop body in HashOrNoop.

During the execution of alpha. Exp(alpha, big.NewInt(int64(i)), nil), alpha will be set to 1 (as i = 0 in the first iteration) and will remain 1 for all future iterations.

Impact This changes the computation of the function to effectively sum(input), which does not appear to be the desired computation.

Recommendation Change alpha.Exp(...) to new(big.Int).Exp(...), or remove this logic if unused.

4.1.11 V-SCT-VUL-011: Bitpacking operation may lead to overlap

Severity	Low	Commit	89b5a01
Type	Logic Error	Status	Fixed
File(s)	poseidon/bn254.go		
Location(s)	HashOrNoop()		
Confirmed Fix At	5766879		

In HashOrNoop, if there are fewer than 4 elements in the input vector (len(input) <= 3), the function attempts to pack the elements into a single BN254 variable and return the packed value otherwise unmodified:

```
if len(input) <= 3 {
        returnVal := frontend.Variable(0)

alpha := new(big.Int).SetInt64(1 << 32)
for i, inputElement := range input {
        returnVal = c.api.MulAcc(returnVal, inputElement, alpha.Exp(alpha, big.NewInt (int64(i)), nil))
}
return BN254HashOut(returnVal)
} else { ... }</pre>
```

Snippet 4.14: Case in HashOrNoop when len(input) <= 3.</pre>

(Note that this does not function correctly due to the prior issue V-SCT-VUL-010).

The value of alpha is 1 << 32; however, Goldilocks variables may take up 64 bits, and so the packing operation may end up modifying some of the bits of each of the input elements while constructing the output value. This logic appears to be implemented correctly in HashNoPad, which performs this operation with two_to_64:

```
two_to_32 := new(big.Int).SetInt64(1 << 32)
two_to_64 := new(big.Int).Mul(two_to_32, two_to_32)
...
inter := frontend.Variable(0)
for k := 0; k < len(bn254Chunk); k++ {
    inter = c.api.MulAcc(inter, bn254Chunk[k].Limb, new(big.Int).Exp(two_to_64, big.NewInt(int64(k)), nil))
}</pre>
```

Snippet 4.15: Bitpacking operation in HashNoPad.

Impact This changes the computation of the function, as the function does perform computation over the input values and is not a no-op as the function name implies.

Recommendation Change alpha to 1 << 64 (see HashNoPad for computation of two_to_64), or remove this logic if unused.

Developer Response We applied the recommended fix by replacing alpha := $(1 \ll 32)$ with alpha := $(1 \ll 32)^2$.

4.1.12 V-SCT-VUL-012: evalL0 overconstrained

Severity	Low		Commit	89b5a01
Type	Overconstrained Circuit		Status	Fixed
File(s)	plonk/plonk.go			
Location(s)	evalL0()			
Confirmed Fix At	d241f54			

The evalL0 function evaluates $(x^n - 1) / (n * (x-1))$.

```
denominator := glApi.SubExtension(
    glApi.ScalarMulExtension(x, p.DEGREE),
    p.DEGREE_QE,

    return glApi.DivExtension(
        evalZeroPoly,
        denominator,
    )
```

Snippet 4.16: Computation of and division by n * (x - 1), computed as n * x - n.

As described in V-SCT-VUL-008, DivExtension() is a *safe* division, asserting that denominator != 0. If x happens to equal 1, then the denominator will be zero, and these constraints will be unsatisfiable.

Note that 0xPolygonZero's plonky2 repository handles the x = 1 case by returning 1 directly.

Impact If the evaluation point ever happens to be 1, the circuits will become unsatisfiable. In this case, the prover will need to generate a new proof, likely going through the grinding process again to generate new challenges.

Recommendation As the uses of DivExtension() in this issue and in V-SCT-VUL-008 are the only (non-test) uses of DivExtension(), consider making a function DivExtensionOr(), which returns the division when the denominator is non-zero and a default value otherwise.

Developer Response [If applicable]

4.1.13 V-SCT-VUL-013: Salted evaluation not considered

Severity	Low		Commit	89b5a01
Type	Logic Error		Status	Fixed
File(s)	fri/fri.go			
Location(s)	friCombineInitial()			
Confirmed Fix At				

In friCombineInitial(), the oracle evaluations of the polynomials from each batch are gathered in a loop.

```
for i := 0; i < len(instance.Batches); i++ {</pre>
1
           batch := instance.Batches[i]
3
           reducedOpenings := precomputedReducedEval[i]
4
           point := batch.Point
           evals := make([]gl.QuadraticExtensionVariable, 0)
6
7
           for _, polynomial := range batch.Polynomials {
8
               evals = append(
9
                   evals,
                    gl.QuadraticExtensionVariable{
10
                        proof.EvalsProofs[polynomial.OracleIndex].Elements[polynomial.
11
       PolynomialInfo],
                        gl.Zero(),
12
13
                   },
               )
14
           }
15
```

Snippet 4.17: Snippet from friCombineInitial().

In the corresponding location in the plonky2 codebase, a variable salted is computed and the evaluation is computed using unsalted_eval().

```
let poly_blinding = instance.oracles[p.oracle_index].blinding;
let salted = params.hiding && poly_blinding;
proof.unsalted_eval(p.oracle_index, p.polynomial_index, salted)
```

Snippet 4.18: Computation of each element of evals in the plonky2 repository.

The definition of unsalted_eval() is shown below. salt_size(b) is equivalent to the statement if b SALT_SIZE else 0.

```
pub(crate) fn unsalted_eval(&self, oracle_index: usize, poly_index: usize, salted:
    bool) -> F {
    self.unsalted_evals(oracle_index, salted)[poly_index]
}

fn unsalted_evals(&self, oracle_index: usize, salted: bool) -> &[F] {
    let evals = &self.evals_proofs[oracle_index].0;
    &evals[..evals.len() - salt_size(salted)]
}
```

Snippet 4.19: Definition of unsalted_eval()

As can be seen, the Go implementation assumes salted is set to false. Since each plonk oracle in the Go repository does have a Blinding parameter which may be true or false, this equates to hard-coding params. Hiding to false.

However, the params. Hiding attribute is used elsewhere in the codebase, e.g. in validateFriProofShape (), so it is not assumed to be false everywhere.

Impact Extra work may occur to validate the salt evaluations. A prover working based off of the plonky2 repository may incorrectly elide these evaluations.

Recommendation Implement the logic of unsalted_eval().

Developer Response We have decided to require the Hiding parameter to be set to false.

4.1.14 V-SCT-VUL-014: Index calculation differs from reference implementation

Severity	Low	Commit	89b5a01
Type	Logic Error	Status	Fixed
File(s)	plonk/gates/coset_interpolation_gate.go		
Location(s)	EvalUnfiltered()		
Confirmed Fix At	5dd6da2		

The calculation of endIndex in function EvalUnfiltered of the CosetInterpolationGate deviates from the reference implementation. Specifically, the reference implementation takes the min between start_index + degree - 1 and num_points (see here). The gnark implementation, on the other hand, does not consider the number of points at all (see snippet bellow).

```
1 | endIndex := startIndex + g.degree - 1
```

Snippet 4.20: Snippet from EvalUnfiltered().

Impact This may result in a crash due to out of bounds index or, worse, wrong constraint evaluation.

Recommendation We recommend making the gnark implementation consistent with the reference one.

4.1.15 V-SCT-VUL-015: Contracts: _callbackGasLimit is unused

Severity	Low	Commit	abd4356
Type	Denial of Service	Status	Fixed
File(s)	contracts/src/SuccinctGateway.sol		
Location(s)	fulfillCallback()		
Confirmed Fix At	https://github.com/succinctlabs/succinctx/pull/316		

The SuccinctGateway contract contains a fulfillCallback() function that the prover calls to fulfill an earlier request made via the requestCallback() function.

```
1 /// @dev Fulfills a request by providing the output and proof.
2 /// @param _nonce The nonce of the request.
3 /// @param _functionId The function identifier.
4 /// @param _inputHash The hash of the function input.
5 /// @param _callbackAddress The address of the callback contract.
6 /// @param _callbackSelector The selector of the callback function.
7 /// @param _callbackGasLimit The gas limit for the callback function.
8 /// @param _context The function context.
9 /// @param _output The function output.
10 /// @param _proof The function proof.
11 function fulfillCallback(
```

Snippet 4.21: Definition of SuccinctGateway.fulfillCallback()

The _callbackGasLimit parameter of the fulfillCallback() function is not used within that function to limit the gas usage of the callback function.

Impact The callback function can contain arbitrary code that may have a very high gas usage or exhaust the gas limit altogether and cause the callback to revert. If the provers are expected to pay the gas fees, a malicious callback function may attack their solvency.

Recommendation If the intention is for the user to pay the callback gas fees, and for the provers to pay the fees of the fulfillCallback() function itself, the callback gas should be limited by using .call{gas: _callbackGasLimit}(...).

Note that, in this case, there is a still the opportunity for a "returnbomb," in which the user callback returns a very large value. Though the fulfillCallback() function does not use the return value, the generated EVM code will copy the entire returndata to memory after the call. To mitigate this, inline assembly can be used instead of the Solidity call() function so that the return data can be ignored altogether. See https://github.com/nomad-xyz/ExcessivelySafeCall for a more complete description and solution.

Developer Response Although we generally will always simulate before sending for fulfill *() transactions, we added {gas: _callbackGasLimit} to the _callbackAddress.call() statement as an extra precaution.

4.1.16 V-SCT-VUL-016: 1 added to TwoAdicSubgroup output twice

Severity	Warning	Commit	89b5a01
Type	Logic Error	Status	Fixed
File(s)	goldilocks/base.go		
Location(s)	TwoAdicSubgroup()		
Confirmed Fix At	2043890		

1 is added to the output vector res before the for loop, which then computes the remainder of the subgroup with the primitive root of unity (i.e., ω , ω^2 ,...).

```
res = append(res, goldilocks.NewElement(1))

for i := 0; i < (1 << nLog); i++ {
    lastElement := res[len(res)-1]
    res = append(res, *lastElement.Mul(&lastElement, &rootOfUnity))
}</pre>
```

Snippet 4.22: Loop computing the two-adic subgroup in TwoAdicSubgroup.

However, the loop iterates through values $[0, 2^{nLog} - 1]$, which compute output values ranging from $[\omega, \omega^{2^{nLog}}]$. Since $\omega^{2^{nLog}} = 1$, the last element added to the res vector will be 1, which is redundant, as 1 was already added to the vector.

Impact The output vector will contain the element 1 twice (i.e., $[1, \omega, \omega^2, \dots, \omega^{2^{nLog-1}}, 1]$).

Recommendation Change the loop iteration to be bounded on i < (1 << nLog) - 1.

4.1.17 V-SCT-VUL-017: Missed opportunities to use appropriate abstractions

Severity	Warning	Commit	89b5a01
Type	Maintainability	Status	Fixed
File(s)	See issue description		
Location(s)	See issue description		
Confirmed Fix At	9617141		

The following locations use inlined expressions instead of their already-created abstractions:

- ▶ challenger/challenger.go:
 - The declaration of spongeState [poseidon.SPONGE_WIDTH]gl.Variable can be replaced with spongeState gl.GoldilocksState.
- ▶ goldilocks/base.go:
 - Functions Sub and SubNoReduce can replace MODULUS.Uint64()-1 with NegOne().
 - Function Reduce duplicates the logic of ReduceWithMaxBits, so the body of Reduce could be replaced with a single call: ReduceWithMaxBits(x, RANGE_CHECK_NB_BITS).
- ▶ goldilocks/quadratic_extension.go:
 - Function MulExtensionNoReduce could have replaced 7 with W.
 - Function InverseExtension could have replaced p.api.IsZero(...Limb) with p. IsZero(...).
 - Functions MulAddExtension, MulExtension, SubMulExtension could have replaced two calls to Reduce with a single call to ReduceExtension.
 - Functions AddExtension and SubExtension could replace their calls to Add/Sub to AddExtensionNoReduce/SubExtensionNoReduce followed by a call to ReduceExtension.
- goldilocks/quadratic_extension_algebra.go:
 - Function ToQuadraticExtensionAlgebra could have replaced 2 with D.
- ▶ poseidon/goldilocks.go:
 - Function HashNoPad could have replaced number 4 with len(hash).
 - Function constantLayer could have replaced c.gl.MulAdd(..., gl.NewVariable(1), ...) with c.gl.Add(..., ...).
 - Functions mdsRowShf and MdsRowShfExtension could have replaced [SPONGE_WIDTH]gl
 .Variable with GoldilocksState.
 - Function mdsPartialLayerInit could replace gl.NewVariable(0) with a call to gl. Zero().
- ▶ plonk/gates/reducing_gate.go:
 - In function EvalUnfiltered, coeff could be initialized with ToQuadraticExtensionAlgebra (ToQuadraticExtension(NewVariable(coeffs[i]))) rather than looping over D. A similar issue occurs in numerous locations.

Impact This inlining makes the code slightly more verbose, and if the internal structure changes (e.g. from refactoring), it would be easier and less error-prone to make one change to the abstraction rather than at all inlined sites.

Recommendation Use the appropriate abstractions.

4.1.18 V-SCT-VUL-018: Unused functions/variables

Severity	Warning	Commit	89b5a01
Type	Maintainability	Status	Fixed
File(s)	See issue description		
Location(s)	See issue description		
Confirmed Fix At	1a03726		

A few functions and variables are defined, but not used in the codebase.

- ▶ goldilocks/base.go:
 - var REDUCE_NB_BITS_THRESHOLD uint8 is unused.
 - func (p *Chip) Exp(...) is unused.
- ▶ poseidon/goldilocks.go:
 - const MAX_WIDTH is unused.
- ► types/utils.go:
 - func ReductionArityBits() is unused.
- ▶ variables/plonk.go
 - func NewOpeningSet(...) is unused.

Impact If unmaintained, these functions may become out-of-sync with the codebase and produce issues down the road if used.

Recommendation Remove the unused values.

4.1.19 V-SCT-VUL-019: Bit reversal incorrect for ArityBits != 4

Severity	Warning		Commit	89b5a01
Type	Logic Error		Status	Fixed
File(s)	fri/fri.go			
Location(s)	computeEvaluation()			
Confirmed Fix At			2fab6a9	

The computeEvaluation() function uses an arityBits of at most 8 to evaluate the next reduced FRI polynomial. evals, which is used to perform this computation, is an array of length arity := 2^arityBits. When permuting the evaluations, 8-bit indices are used.

```
permutedEvals := make([]gl.QuadraticExtensionVariable, len(evals))

for i := uint8(0); i < uint8(len(evals)); i++ {
    newIndex := bits.Reverse8(i) >> arityBits
    permutedEvals[newIndex] = evals[i]
}
```

Snippet 4.23: Evaluation of permutedEvals.

The reversed bits should be shifted by 8 - arityBits, not arityBits. For example, if arityBits is 8, this will always return 0.

Impact For arityBits != 4, the wrong indices will be used. Currently, the only callers use arityBits == 4, so this will only be an issue in future iterations of the protocol.

Recommendation Shift by 8 - arityBits rather than arityBits.

4.1.20 V-SCT-VUL-020: clearBuffer doesn't clear the buffer

Severity	Warning	Commit	89b5a01
Type	Logic Error	Status	Fixed
File(s)	challenger/challenger.go		
Location(s)	clearBuffer()		
Confirmed Fix At		0f6466c	

Function clearBuffer does not perform the expected operation of clearing a provided buffer; rather, it ignores its input argument and instead returns a new buffer.

```
func clearBuffer(buffer []gl.Variable) []gl.Variable {
   return make([]gl.Variable, 0)
}
```

Snippet 4.24: Definition of function clearBuffer.

Impact This has the potential to cause programming errors and confusion about how and when the buffers are cleared.

See related issue V-SCT-VUL-021.

Recommendation Either fix the implementation of clearBuffer so that it clears the buffer provided as an argument or rename the function to newBuffer and remove the unused parameter.

Developer Response We removed the clearBuffer() function to increase the code clarity.

4.1.21 V-SCT-VUL-021: Possible leakage of randomness

Severity	Warning	Commit	89b5a01
Type	Logic Error	Status	Fixed
File(s)	challenger/challenger.go		
Location(s)	duplexing()		
Confirmed Fix At		0f6466c	

The incorrect behavior of the clearBuffer function (see V-SCT-VUL-020) leads to an unintended consequence in the duplexing function, which leaves the outputBuffer unmodified when it was intended to be cleared before adding new data:

Snippet 4.25: Operations on outputBuffer in the duplexing function.

In the current implementation, the outputBuffer will already be empty here due to the proper clearing operation performed in ObserveElement (c.outputBuffer = clearBuffer(c.outputBuffer)).

Impact If the API were to change or the implementation of ObserveElement were to change, the outputBuffer wouldn't be properly cleared in the duplexing function. This could lead to an adaptive attack. The next challenge value would not be dependent on the most recent input, and an attacker could leverage knowledge of the previous outputs to construct proof values that would pass without being an otherwise legitimate proof.

Recommendation Add an assertion into the duplexing function that ensures that the outputBuffer is empty before appending newly computed values to ensure Fiat-Shamir is performed safely.

4.1.22 V-SCT-VUL-022: Unused challenger parameter due to relocated logic

Severity	Warning	Commit	89b5a01
Type	Maintainability	Status	Fixed
File(s)	challenger/challenger.go		
Location(s)	GetFriChallenges()		
Confirmed Fix At		30d73da	

In the function GetFriChallenges, the parameter degreeBits is unused. This is because the FRI query index computation has been moved into the verifyQueryRound function in fri/fri.go.

Snippet 4.26: FRI index computation.

This is a change from the original plonky2 implementation, which performs the computation in the fri_challenges function (the analogue of the GetFriChallenges function).

Impact This could cause confusion for developers and lead to the introduction of redundant computation if someone believes this computation has been erroneously omitted.

Recommendation Remove the unused parameter and add documentation explaining where the logic is performed instead.

4.1.23 V-SCT-VUL-023: Errors in gate Id functions

Severity	Warning	Commit	89b5a01
Type	Data Validation	Status	Fixed
File(s)	See issue description		
Location(s)	Id()		
Confirmed Fix At		cc064ae	

The following gates generate the incorrect Id with their Id functions:

- ▶ base_sum_gate.go:
 - Parameter should be num_limbs not num_ops.
- multiplication_extension_gate.go:
 - Id is listed as ArithmeticExtensionGate instead of MulExtensionGate.
- ▶ reducing_extension_gate.go:
 - Parameter should be num_coeffs not num_ops.
- ▶ reducing_gate.go:
 - Id is listed as ReducingExtensionGate instead of ReducingGate.
 - Parameter should be num_coeffs not num_ops.

Impact These errors result in gate Ids being misreported as the incorrect gate or a gate with mismatched parameters.

Recommendation Ensure that gate Id functions are up to date.

4.1.24 V-SCT-VUL-024: Use of incorrect constant

Severity	Warning	Commit	89b5a01
Type	Maintainability	Status	Fixed
File(s)	fri/fri.go		
Location(s)	verifyMerkleProofToCapWithCapIndex()		
Confirmed Fix At		318c3ce	

At the end of verifyMerkleProofToCapWithCapIndex(), a series of lookups are used to identify the correct Merkle cap.

```
1 // We assume that the cap_height is 4. Create two levels of the Lookup2 circuit
2 if len(capIndexBits) != 4 || len(merkleCap) != 16 {
       // [VERIDISE] ... error message here
3
4 }
6 const NUM_LEAF_LOOKUPS = 4
7 | var leafLookups [NUM_LEAF_LOOKUPS]poseidon.BN254HashOut
  // First create the "leaf" lookup2 circuits
  // The will use the least significant bits of the capIndexBits array
10 | for i := 0; i < NUM_LEAF_LOOKUPS; i++ {</pre>
       leafLookups[i] = f.api.Lookup2(
11
12
           capIndexBits[0], capIndexBits[1],
           merkleCap[i*NUM_LEAF_LOOKUPS], merkleCap[i*NUM_LEAF_LOOKUPS+1], merkleCap[i*
13
      NUM_LEAF_LOOKUPS+2], merkleCap[i*NUM_LEAF_LOOKUPS+3],
14
15 }
```

Snippet 4.27: Lookup assumptions and first round of lookups to find the correct Merkle cap.

Note the arguments passed to Lookup2 are merkleCap[i*NUM_LEAF_LOOKUPS] through merkleCap[i*NUM_LEAF_LOOKUPS+31.

However, the loop stride over merkleCap is a fixed constant determined by Lookup2: 4. If NUM_LEAF_LOOKUPS changes to a non-4 value, this loop will be incorrect.

Impact If len(merkleCaps) were some other power of 4 (e.g. 4 or 64), then NUM_LEAF_LOOKUPS would also need to be changed (e.g. to 1 or 16). In this case, the parameters passed to Lookup2() would be incorrect.

For example, if len(merkleCaps) were changed to 64, then the indexing operations would provoke an out-of-bounds error.

Recommendation Access merkleCap on the range 4*i..4*i+3.

Developer Response We applied the recommended fix, using a new constant STRIDE_LENGTH = 4.

4.1.25 V-SCT-VUL-025: Missing degree check for CosetInterpolationGate

Severity	Warning	Commit	89b5a01
Type	Data Validation	Status	Fixed
File(s)	plonk/gates/coset_interpolation_gate.go		
Location(s)	deserializeCosetInterpolationGate()		
Confirmed Fix At		888b247	

In deserializeCosetInterpolationGate, the parsed value for degree is not checked to be within a certain range. However, according to the reference implementation, the degree needs to be at least 2 in order for the coset interpolation to function properly.

Impact The missing check could lead to a divide-by-zero error in numIntermediates() if g.degree == 1 or invalid computation if g.degree == 0.

Recommendation Add an assertion to check that degreeInt > 1in deserializeCosetInterpolationGate

4.1.26 V-SCT-VUL-026: Contracts: Centralization Risk

Severity	Warning	Commit	abd4356
Type	Access Control	Status	Partially Fixed
File(s)	contracts/src/SuccinctGateway.sol		
Location(s)	SuccinctGateway		
Confirmed Fix At	https://github.com/succinctlabs/succinctx/pull/310		

The SuccinctGateway contract has a special set of accounts (called guardians) which can perform privileged actions. This includes two primary actions:

- 1. Setting the feeVault address.
- 2. Adding to/removing from the whitelisted allowedProvers list.

A second privileged role (called the TIMELOCK_ROLE) is given the ability to upgrade the contract.

Impact While the centralization risk associated to the TIMELOCK_ROLE is reduced by assigning the role to a time-locked contract, if any guardian's keys are compromised users may be harmed in the following ways:

- 1. The fee vault address may be changed, stealing funds from the manager of the contract.
- 2. A malicious prover may use the fulfillCall() functions to make arbitrary calls on the contract's behalf.

Recommendation To mitigate the risks above, developers should take actions such as:

- ▶ Always use the SuccinctGateway contract with a multi-signature wallet acting as its owner.
- ► Actively monitor the current guardian set.
- ▶ Introduce a time delay before key operations are performed (such as changing the allowedProvers).

Developer Response We have mitigated this by allowing individual users to manage the whitelisted sets of provers for the functions which they own, if they prefer a custom prover set to the ones provided by the guardians.

4.1.27 V-SCT-VUL-027: Contracts: All _callbackAddresses permitted

Severity	Warning	Commit	abd4356
Type	Data Validation	Status	Fixed
File(s)	contracts/src/SuccinctGateway.sol		
Location(s)	fulfillCall(), fulfillCallback()		
Confirmed Fix At	https://github.com/succinctlabs/succinctx/pull/319		

The fulfillCall() and fulfillCallback() functions are used by provers to complete submitted requests once verification has passed.

Snippet 4.28: Snippet from fulfillCallback() invoking the user-requested _callbackAddress.

While requestCallback() endpoint sets the $_$ callbackAddress to msg.sender, a requestCall may request a call to any address.

Further, for many reasons, a fulfillCallback() may be invoked on an unexpected address. For example

- 1. An error in off-chain prover code may lead to calling the wrong address.
- 2. A malicious prover (see V-SCT-VUL-026) may invoke arbitrary addresses/functions.
- 3. A user may set the _entryAddress of a requestCall() to the address of the SuccinctGateway itself, causing the SuccinctGateway contract itself to call requestCallback().

Impact Depending on future features added to the contract, its future holdings in other on-chain assets, or assumptions other contracts make about the SuccinctGateway's behavior, unexpected behavior may occur.

Note that the impact is limited by the fact that these callbacks cannot send ether, and under normal operation conditions no other digital assets are owned by the contract.

Recommendation Allow whitelisting or blacklisting of callback addresses.

Developer Response We have made the contracts non-upgradeable.

4.1.28 V-SCT-VUL-028: Replace GoldilocksHashOut size with named constant

Severity	Info	Commit	89b5a01
Type	Maintainability	Status	Fixed
File(s)	poseidon/goldilocks.g	o, challenger	/challenger.go
Location(s)	type GoldilocksHashOut, GetHash()		
Confirmed Fix At		7c7a01a	

The definition of GoldilocksHashOut does not use a named constant value to define the length of the underlying variable array.

Impact This leads to minor maintainability issues, such as having to propagate a hard-coded constant to other locations, such as GetHash() in challenger.go:

```
func (c *Chip) GetHash() poseidon.GoldilocksHashOut {
    return [4]gl.Variable{c.GetChallenge(), c.GetChallenge(), c.GetChallenge(), c.GetChallenge()}

3 }
```

Recommendation Use a named constant value to refer to the length of the GoldilocksHashOut type.

4.1.29 V-SCT-VUL-029: Type error in bn254's HashOrNoop()

Severity	Info		Commit	89b5a01
Type	Maintainability		Status	Fixed
File(s)	poseidon/bn254.go			
Location(s)	HashOrNoop()			
Confirmed Fix At			40d71e9	

In HashOrNoop, the inputElement argument to api.MulAcc has an incorrect type. api.MulAcc expects a frontend.Variable, but inputElement has type goldilocks.Variable.

```
func (c *BN254Chip) HashOrNoop(input []gl.Variable) BN254HashOut {
2
       if len(input) <= 3 {</pre>
           returnVal := frontend.Variable(0)
3
4
           alpha := new(big.Int).SetInt64(1 << 32)</pre>
           for i, inputElement := range input {
6
                returnVal = c.api.MulAcc(returnVal, inputElement, alpha.Exp(alpha, big.
7
       NewInt(int64(i)), nil))
8
           }
9
           return BN254HashOut(returnVal)
10
11
       } else {
12
           return c.HashNoPad(input)
       }
13
14 }
```

Snippet 4.29: Snippet from poseidon/bn254.go

Impact This results in a circuit compilation error whenever input has length less than or equal to 3.

```
parsing circuit error="
goldilocks.Variable to big.Int not supported
frontend.parseCircuit.func2
compile.go:118
utils.FromInterface
convert.go:86
```

Recommendation Modify inputElement to inputElement.Limb.

4.1.30 V-SCT-VUL-030: Non-std range-check used

Severity	Info	Commit	89b5a01
Type	Maintainability	Status	Fixed
File(s)	goldilocks/base.go		
Location(s)	type Chip		
Confirmed Fix At	https:		
	//github.com/succinctlabs/gnark-plonky2-verifier/pull/47		

The goldilocks base Chip uses frontend. Rangechecker for range checks.

Snippet 4.30: Type declaration of goldilocks Chip.

The Gnark documentation for frontend. Rangechecker recommends:

Users should instead use github.com/consensys/gnark/std/rangecheck package which automatically chooses most optimal method for range checking the variables.

Impact Certain compilers may not support this implementation.

Recommendation Use std.rangecheck instead of frontend.Rangechecker.

Developer Response We applied the recommended fix.

Updated Veridise Response Due to issue https://github.com/advisories/GHSA-rjjm-x32p-m3f7 (see also https://github.com/Consensys/gnark/issues/897 and https://github.com/Consensys/gnark/com we additionally recommend that the developers either wait for gnark version 0.9.2 or manually verify that this will not be an issue by adding additional checks at circuit compilation time.

Updated Developer Response We have added additional checks to ensure that the comparison performed is precise, even in the presence of the referenced issue.

4.1.31 V-SCT-VUL-031: Various out-of-date comments and documentation

Severity	Info	Commit	89b5a01
Type	Maintainability	Status	Fixed
File(s)	See issue description		
Location(s)	S	ee issue description	
Confirmed Fix At		f256ca6	

The following locations have out-of-date comments and documentation:

- ► fri/fri.go: Comment in assertLeadingZeros references non-existent variables from the reference implementation (leading_zeros; should instead reference ProofOfWorkBits).
- ▶ go.mod: Says that the required go version is 1.19, but the README says to use go version 1.20.1+.
- goldilocks/base.go: the comments of functions Add, AddNoReduce, Sub, SubNoReduce, Mul, MulNoReduce, MulAdd, MulAddNoReduce are out of date, and some are incorrect (argument names mismatched, incorrect arguments used, etc.).
- ▶ goldilocks/quadratic_extensions.go: the documentation claims that the function SubMulExtension constrains a, b, c such that res = a*b c. However, the function introduces constraints for res = (a b)*c.
- ▶ goldilocks/quadratic_extensions.go: the documentation claims that the function Lookup2 is similar to gnark's Select2 and that it operates on a variable. However, the function in gnark is actually also called Lookup2, and it operates on two variables.
- ▶ plonk/gates/reducing_extension_gate.go: the comment in deserializeReducingExtensionGate is copied from ReducingGate.
- ▶ plonk/gates/random_access_gate.go: lines 153-154, the comment claims that the added constrained are bx (by y) but the implementation adds by (bx x).

Impact This can introduce bugs in future versions of the codebase when new developers mistakenly use the function for what the documentation is claiming.

Recommendation Update the documentation to reflect current the implementation.

4.1.32 V-SCT-VUL-032: Typos in code

Severity	Info	Commit	89b5a01
Type	Maintainability	Status	Fixed
File(s)	See issue description		
Location(s)	See is	sue description	
Confirmed Fix At		de0ff4f	

The codebase contains a few minor typos, listed below.

- ► zetaNextBath in fri/fri.go.
- ▶ If the bit is on (should be "one") in fri/fri.go.
- ▶ panic("len(evals) ! arity") (should be "!=") in fri/fri.go.
- ► aritheticExtensionGateRegex in plonk/gates/arithmetic_extension_gate.go.
- ▶ aritheticGateRegex in plonk/gates/arithmetic_extension.go.

4.1.33 V-SCT-VUL-033: Ignored gate parameters

Severity	Info	Commit	89b5a01
Type	Maintainability	Status	Fixed
File(s)	<pre>plonk/gates/{exponentiation_gate.go,</pre>		
	random_access_gate.go}		
Location(s)	exponentiationGateRegex, randomAccessGateRegex		
Confirmed Fix At		06f91e4	

Both the ExponentiationGate and RandomAccessGate capture a base parameter (corresponding to constant D) while describilizing the gate parameters. This parameter is then unused.

Snippet 4.31: Example regex from exponentiation_gate.go.

Impact These ignored parameters may become an issue down the road if they no longer match the assumptions in the codebase.

Recommendation Either check that parameters["base"] matches the current assumptions of the codebase (i.e., parameters["base"] == gl.D), or add documentation explaining why checking parameters["base"] is unnecessary.

4.1.34 V-SCT-VUL-034: Hard-coded constants in code

Severity	Info		Commit	89b5a01
Type	Maintainability		Status	Fixed
File(s)	See issue description			
Location(s)	See issue description			
Confirmed Fix At			e3eff27	

Certain parts of the code use hard-coded values rather than relying on global constants.

- plonk/gates/poseidon_mds_gate.go
 - In EvalUnfiltered(), a quadratic extension variable is constrained to be zero.

```
1 diff := glApi.SubExtensionAlgebra(output, computed_outputs[i])
2 constraints = append(constraints, diff[0], diff[1])
```

Snippet 4.32: Snippet from EvalUnfiltered()

If the extension degree is ever changed to be larger than 2, this will only constrain the first two components. As implemented elsewhere in the code, the preferred way to constrain a QuadraticExtensionVariable to be zero is to use a loop with bound gl.D.

- plonk/gates/public_input_gate.go
 - In EvalUnfiltered(), a loop uses a hardcoded constant for iterating over wires and hash values rather than iterating over the total number of wires (i.e., len(wires)). This could be problematic if the gate is ever modified to take a different number of input wires.

Snippet 4.33: Snippet from EvalUnfiltered()

Impact If these constants change in the future, these implementations will become incorrect.

Recommendation Use the applicable constant.

4.1.35 V-SCT-VUL-035: No assertion that gnark is using BN254

Severity	Info	Commit	89b5a01
Type	Maintainability	Status	Fixed
File(s)	poseidon/bn254.go		
Location(s)	New	BN254Chip()	
Confirmed Fix At		4951161	

The BN254 Poseidon chip computes the Poseidon hash function over the BN254 scalar field. The implementation assumes the underlying field is the scalar field for BN254.

However, gnark supports several other fields, any of which may be used when compiling the constraints.

Impact If the incorrect field is used, the BN254 chip will no longer be computing a Poseidon hash over the BN254 field.

Recommendation Consider adding a check to NewBN254Chip() to ensure the api.Compiler(). Field() is as expected.

4.1.36 V-SCT-VUL-036: Inaccurate bounds-check on numConsts

Severity	Info		Commit	89b5a01
Type	Data Validation		Status	Fixed
File(s)	plonk/gates/constant_gate.go			
Location(s)		ConstInp	ıt(), WireOutpu	t()
Confirmed Fix At			9e6d08b	

In both the ConstInput and WireOutput function, the input argument is checked to ensure it is not greater than the number of constants.

```
func (g *ConstantGate) ConstInput(i uint64) uint64 {
    if i > g.numConsts {
        panic("Invalid constant index")
    }
    return i
6 }
```

Snippet 4.34: Definition of ConstInput().

However, since the return value is used as an array index into an array of size g.numConsts, i should be checked to ensure it is not greater than *or equal* to g.numConsts (i.e., if $i \ge g$. numConsts).

Impact This could lead to an array out of bounds error if i == g.numConsts.

Recommendation Update the check on i to be $i \ge g.numConsts$.

4.1.37 V-SCT-VUL-037: Sub-optimal sub-expression in random access gate

Severity	Info	Commit	89b5a01
Type	Constraint Optimization	Status	Fixed
File(s)	plonk/gates/random_access_gate.go		
Location(s)	EvalUnfiltered()		
Confirmed Fix At		8567f33	

The EvalUnfiltered function of the random access gate creates sub-optimal expressions while building the constraints. Specifically, for every two consecutive elements (x, y) in slice listItems, EvalUnfiltered creates the expression by - (bx - x). However, this can be simplified to x + b(y - x), which has one fewer operation.

```
mul1 := glApi.MulExtension(b, x)
sub1 := glApi.SubExtension(mul1, x)

mul2 := glApi.MulExtension(b, y)
sub2 := glApi.SubExtension(mul2, sub1)

listItemsTmp = append(listItemsTmp, sub2)
```

Snippet 4.35: Snippet from EvalUnfiltered

Impact Since every sub-expression will add several constraints, this logic will yield a sub-optimal overall circuit.

Recommendation We recommend simplifying the sub-expressions to x + b(y - x).

4.1.38 V-SCT-VUL-038: Contracts: Code Recommendations

Severity	Info	Commit	abd4356
Type	Maintainability	Status	Fixed
File(s)	contracts/src/FunctionRegistry.sol		
Location(s)	FunctionRegistry		
Confirmed Fix At	https://github.com/succinctlabs/succinctx/pull/315;https:		
	//github.com/succinctlabs/succinctx/pull/314		

- 1. FunctionRegistry implements the IFunctionRegistry interface, but does not mark the functions as override.
- 2. FunctionRegistry contains several functions with similar functionality. For instance, registerFunction() registers a function, while deployAndRegisterFunction() first deploys a contract, and then registers it. As can be seen in the below snippet, the functions are nearly identical.

```
functionId = getFunctionId(_owner, _salt);
if (address(verifiers[functionId]) != address(0)) {
    revert FunctionAlreadyRegistered(functionId); // should call update instead
}
if (_verifier == address(0)) {
    revert VerifierCannotBeZero();
}
verifierOwners[functionId] = _owner;
verifiers[functionId] = _verifier;
emit FunctionRegistered(functionId, _verifier, _salt, _owner);
```

Snippet 4.36: Definition of registerFunction()

```
functionId = getFunctionId(_owner, _salt);
if (address(verifiers[functionId]) != address(0)) {
    revert FunctionAlreadyRegistered(functionId); // should call update instead
}

verifierOwners[functionId] = _owner;
verifier = _deploy(_bytecode, functionId);
verifiers[functionId] = verifier;

emit FunctionRegistered(functionId, verifier, _salt, _owner);
```

Snippet 4.37: Definition of deployAndRegisterFunction()

There is similar duplication between updateFunction() and deployAndUpdateFunction().

Impact

- 1. If a function is ever deleted from the interface, the corresponding change may be forgotten in the FunctionRegistry contract.
- 2. Changes to the code may be made in only one function, leading to errors or inconsistencies down the road.

Recommendation Label the functions as overridden and remove the code duplication.

Developer Response

- 1. Functions marked as override in SuccinctGateway and FunctionRegistry.
- 2. Common functionality moved into internal register() and update() functions.

4.1.39 V-SCT-VUL-039: Contracts: Possible Wasted Gas

Severity	Info	Commit	abd4356
Type	Gas Optimization	Status	Fixed
File(s)	contracts/src/FunctionRegistry.sol,		
	contracts/src/SuccinctGateway.sol		
Location(s)	See Issue Description		
Confirmed Fix At	https://github.com/succinctlabs/succinctx/pull/313,https:		
	//github.com/succinctlabs/succinctx/pull/315		

When a user submits a transaction to the Ethereum blockchain for processing, they specify the maximum gas fee they are willing to pay. The complexity of the computation in the transaction determines the amount of gas used to process the transaction.

- The Solidity language provides the delete keyword to reset variables to the default value for the data type. Using the delete keyword also refunds a small amount of the gas fee spent on the transaction. The SuccinctGateway.removeProver() function could use delete allowedProvers[_prover] instead of allowedProvers[_prover] = false since the default value for the bool data type is false.
- 2. To save on gas fees in the case where a transaction reverts, it is best for the conditions that could cause a revert to be checked as soon as possible. The FunctionRegistry contract has two functions where the revert-case gas cost could be optimized by checking the simpler _verifier == address(0) condition before computing the functionId.

```
functionId = getFunctionId(_owner, _name);
if (address(verifiers[functionId]) != address(0)) {
    revert FunctionAlreadyRegistered(functionId); // should call update instead
}
if (_verifier == address(0)) {
    revert VerifierCannotBeZero();
}
```

Snippet 4.38: Snippet from FunctionRegistry.registerFunction()

```
functionId = getFunctionId(msg.sender, _name);

if (msg.sender != verifierOwners[functionId]) {
    revert NotFunctionOwner(msg.sender, verifierOwners[functionId]);

if (_verifier == address(0)) {
    revert VerifierCannotBeZero();

}
```

Snippet 4.39: Snippet from FunctionRegistry.updateFunction()

3. The addProver() and removeProver() functions in the SuccinctGateway contract are used to manage the addresses in the allowedProvers map by adding/removing the given address. The presence or absence of address(0) in the allowedProvers map has no meaningful effect. Thus, both functions could allow users to avoid spurious executions before submitting the transaction by reverting when _prover == address(0).

```
function addProver(address _prover) external onlyGuardian {
   allowedProvers[_prover] = true;
   emit ProverUpdated(_prover, true);
}
```

Snippet 4.40: Definition of SuccinctGateway.addProver(). removeProver() is similar.

```
1 | modifier onlyProver() {
2          if (!allowedProvers[msg.sender]) {
3              revert OnlyProver(msg.sender);
4          }
5          _;
6     }
```

Snippet 4.41: Definition of SuccinctGateway.onlyProver() modifier

Impact Higher than necessary gas fees may be required in several functions.

Recommendation

- In SuccinctGateway.removeProver(),replace allowedProvers[_prover] = false with delete allowedProvers[_prover].
- 2. In FunctionRegistry.registerFunction() and FunctionRegistry.updateFunction(), move the _verifier == address(0) check to the start of the function.
- 3. In SuccinctGateway.addProver() and SuccinctGateway.removeProver() add a condition to revert if _prover == address(0).

Developer Response

- 1. We applied recommendation.
- 2. We applied recommendation.
- 3. The addProver() and removeProver() functions were removed from SuccinctGateway in https://github.com/succinctlabs/succinctx/pull/310 and replaced with addCustomProver (), addDefaultProver(), removeCustomProver(), and removeDefaultProver(). However, none of these functions require the condition _prover != address(0).

5.1 Methodology

Veridise auditors set out to fuzz test the witness generation in gnark-plonky2-verifier to increase confidence in its implementation and gain greater assurance in the correctness of constraints which have been determined (by manual review) to match the witness generator. These fuzz tests focused on functional correctness i.e, whether the implementation deviates from the intended behavior.

The auditors primarily performed differential fuzzing—running an alternative implementation on the same inputs and then comparing the results. Some of these alternative implementations were hand-crafted to match the desired behavior, and others were taken from popular repositories such as https://github.com/iden3/circomlib and https://github.com/iden3/go-iden3-crypto. See section (5.3) for more details.

5.2 Properties Fuzzed

Table 5.1 describes the invariants we fuzz-tested. The second column states which component (e.g., correct emulation of addition in the Goldilocks field) the invariant is associated with. The third shows the total amount of compute time spent fuzzing this property. The last column notes whether we found a bug when fuzzing the invariant (X indicates no bug was found and $\sqrt{}$ means fuzzing this invariant revealed a bug).

The Veridise auditors devoted a total of 303 compute-hours to fuzzing this protocol, identifying a total of 0 bugs.

Specification	Invariant	Minutes Fuzzed	Bugs Found
V-SCT-SPEC-001	Spec: base — Add	120	Х
V-SCT-SPEC-002	Spec: base — Exp	120	X
V-SCT-SPEC-003	Spec: base — Inverse	120	X
V-SCT-SPEC-004	Spec: base — Mul	120	X
V-SCT-SPEC-005	Spec: base — MulAdd	120	X
V-SCT-SPEC-006	Spec: base — RangeCheck	60	X
V-SCT-SPEC-007	Spec: base — Reduce	120	X
V-SCT-SPEC-008	Spec: poseidon — bn254	8000	X
V-SCT-SPEC-009	Spec: poseidon — goldilocks	8000	X
V-SCT-SPEC-010	Spec: quadratic_extension — AddExtension	120	X
V-SCT-SPEC-011	Spec: quadratic_extension — ExpExtension	120	X
V-SCT-SPEC-012	Spec: quadratic_extension — InnerProductExten	120	X
V-SCT-SPEC-013	Spec: quadratic_extension — InverseExtension	120	X
V-SCT-SPEC-014	Spec: quadratic_extension — IsZero	120	X
V-SCT-SPEC-015	Spec: quadratic_extension — Lookup	120	X
V-SCT-SPEC-016	Spec: quadratic_extension — Lookup2	120	X

Table 5.1: Invariants Fuzzed.

V-SCT-SPEC-017	Spec: quadratic_extension — MulAddExtension	120	X
V-SCT-SPEC-018	Spec: quadratic_extension — MulExtension	120	X
V-SCT-SPEC-019	Spec: quadratic_extension — ScalarMulExtensio	120	X
V-SCT-SPEC-020	Spec: quadratic_extension — SubExtension	120	X
V-SCT-SPEC-021	Spec: quadratic_extension — SubMulExtension	120	X

5.3 Detailed Description of Fuzzed Specifications

In the rest of this section, *M* denotes the Goldilocks prime.

5.3.1 V-SCT-SPEC-001: Spec: base — Add

Minutes Fuzzed	120	Bugs Found	0

Natural Language We fuzzed the witness generation for the Add() function in goldilocks/base.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of Add satisfies the below formula.

 $Add(x, y) = (x + y) \mod M$.

5.3.2 V-SCT-SPEC-002: Spec: base — Exp

Minutes Fuzzed	120	Bugs Found	0

Natural Language We fuzzed the witness generation for the Exp() function in goldilocks/base.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results. We in particular use a naive exponentiation by squaring to implement the Go function.

Formal Specification We checked the output of Exp satisfies the below formula.

 $\operatorname{Exp}(x, n) = x^n \mod M$.

5.3.3 V-SCT-SPEC-003: Spec: base — Inverse

Minutes Fuzzed 120 Bugs Found 0

Natural Language We fuzzed the witness generation for the Inverse() function in goldilocks /base.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results. We in particular use a naive exponentiation by squaring to implement the Go function.

Formal Specification We checked the output of Inverse satisfies the below formula.

Inverse(x) = y such that $xy \mod M = 1$.

But to actually compute y, we use Fermat's little theorem:

$$x^{M} = x \pmod{M}$$
$$x^{M-2} = x^{-1} \pmod{M}$$

where we use exponentiation by squaring for efficiency.

5.3.4 V-SCT-SPEC-004: Spec: base — Mul

Minutes Fuzzed	120	Bugs Found	0

Natural Language We fuzzed the witness generation for the Mul() function in goldilocks/base.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of Mul satisfies the below formula.

 $Mul(x, y) = (x \cdot y) \mod M$.

5.3.5 V-SCT-SPEC-005: Spec: base — MulAdd

Minutes Fuzzed	120	Bugs Found	0

Natural Language We fuzzed the witness generation for the MulAdd() function in goldilocks/base.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of MulAdd satisfies the below formula.

 $MulAdd(x, y, z) = ((x \cdot y) + z) \mod M.$

5.3.6 V-SCT-SPEC-006: Spec: base — RangeCheck

Minutes Fuzzed	60	Bugs Found	0

Natural Language We fuzzed the witness generation for the RangeCheck() function in goldilocks/base.go. We tested for functional correctness by checking if the predicate is satisfied on various inputs, using a naive comparison as a ground truth.

Formal Specification We checked the RangeCheck is equivalent the following predicate: RangeCheck(x) iff x < M.

5.3.7 V-SCT-SPEC-007: Spec: base — Reduce

Minutes Fuzzed	120	Bugs Found	0

Natural Language We fuzzed the witness generation for the Reduce() function in goldilocks/base.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of Reduce satisfies the below formula.

 $Reduce(x) = x \mod M$

5.3.8 V-SCT-SPEC-008: Spec: poseidon — bn254

	Minutes Fuzzed	8000	Bugs Found	0
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Natural Language We fuzzed the witness generation for the Poseidon() function in poseidon/bn254.go. We tested for functional correctness by computing the expected value using Circomlib's implementation at

https://github.com/iden3/circomlib/blob/cff5ab6288b55ef23602221694a6a38a0239dcc0/circuits/poseidon.circom

and comparing the two results. The Circomlib's template is instantiated with PoseidonEx(3, 4)

5.3.9 V-SCT-SPEC-009: Spec: poseidon — goldilocks

Minutes Fuzzed 8000 Bugs Found 0

Natural Language We fuzzed the witness generation for the Poseidon() function in poseidon /goldilocks.go. We tested for functional correctness by computing the expected value using Go's implementation at

https://github.com/iden3/go-iden3-crypto/blob/3fb23d780c02f41857d62ffa04c3a12912b14761/goldenposeidon/poseidon.go#L101

and comparing the two results. The Go function is manually modified to output 12 elements instead of 4.

5.3.10 V-SCT-SPEC-010: Spec: quadratic_extension — AddExtension

Minutes Fuzzed 120 Bugs Found 0

Natural Language We fuzzed the witness generation for the AddExtension() function in goldilocks/quadratic_extension.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of AddExtension satisfies the below formula.

 $AddExtension(a + b\sqrt{7}, c + d\sqrt{7}) = ((a + c) \mod M) + ((b + d) \mod M)\sqrt{7}.$

5.3.11 V-SCT-SPEC-011: Spec: quadratic_extension — ExpExtension

Minutes Fuzzed	120	Bugs Found	0

Natural Language We fuzzed the witness generation for the ExpExtension() function in goldilocks/quadratic_extension.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results. In particular, we use a naive exponentiation based on repeated squaring to implement the Go function.

Formal Specification We checked the output of ExpExtension satisfies the below formula.

ExpExtension $(a + b\sqrt{7}, n) = (a + b\sqrt{7})^n$

5.3.12 V-SCT-SPEC-012: Spec: quadratic_extension — InnerProductExtension

Minutes Fuzzed 120 Bugs Found

Natural Language We fuzzed the witness generation for the InnerProductExtension() function in goldilocks/quadratic_extension.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

0

Formal Specification We checked the output of InnerProductExtension satisfies the below formula.

InnerProductExtension = f

$$f(c, a + b\sqrt{7}, [\langle x_1 + y_1\sqrt{7}, u_1 + v_1\sqrt{7}\rangle, \ldots]) =$$

$$((a + cx_1u_1 + 7cy_1v_1 + cx_2u_2 + 7cy_2v_2 + \ldots) \mod M) +$$

$$((b + cx_1v_1 + cy_1u_1 + cx_2v_2 + cy_2u_2 + \ldots) \mod M)\sqrt{7}$$

5.3.13 V-SCT-SPEC-013: Spec: quadratic_extension — InverseExtension

Minutes Fuzzed 120 Bugs Found 0

Natural Language We fuzzed the witness generation for the InverseExtension() function in goldilocks/quadratic_extension.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of InverseExtension satisfies the below formula.

InverseExtension $(a + b\sqrt{7}) = c + d\sqrt{7}$ such that $ac + 7cd = 1 \pmod{M} \land ad + bc = 0 \pmod{M}$

But to actually compute $c + d\sqrt{7}$, we follow:

$$\frac{1}{a + b\sqrt{7}} = \frac{1}{a + b\sqrt{7}} \cdot \frac{a - b\sqrt{7}}{a - b\sqrt{7}} = \frac{a - b\sqrt{7}}{a^2 - 7b^2}$$

The inverse of $a^2 - 7b^2$ under M can be found by using Fermat's little theorem:

$$x^{M} = x \pmod{M}$$
$$x^{M-2} = x^{-1} \pmod{M}$$

where we use exponentiation by squaring for efficiency.

Note that a = b = 0 is the only case where the inverse is not defined. $a^2 = 7b^2 \pmod{M}$ doesn't have any other solutions.

5.3.14 V-SCT-SPEC-014: Spec: quadratic_extension — IsZero

Minutes Fuzzed 120 Bugs Found

Natural Language We fuzzed the witness generation for the IsZero() function in goldilocks/quadratic_extension.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of IsZero satisfies the below formula.

IsZero = f where

$$f(a+b\sqrt{7}) = \begin{cases} 1 & \text{if } a = 0 \land b = 0\\ 0 & \text{otherwise} \end{cases}$$

5.3.15 V-SCT-SPEC-015: Spec: quadratic_extension — Lookup

Minutes Fuzzed 120 B

Bugs Found 0

Natural Language We fuzzed the witness generation for the Lookup() function in goldilocks/quadratic_extension.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of Lookup satisfies the below formula.

Lookup = f where

$$f(b, u, v) = \begin{cases} u & \text{if } b = 0 \\ v & \text{if } b = 1 \end{cases}$$

5.3.16 V-SCT-SPEC-016: Spec: quadratic_extension — Lookup2

Minutes Fuzzed

120

Bugs Found 0

Natural Language We fuzzed the witness generation for the Lookup2() function in goldilocks /quadratic_extension.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of Lookup2 satisfies the below formula.

Lookup2 = f where

$$f(b_1, b_2, u, v, x, y) = \begin{cases} u & \text{if } b_1 = 0 \land b_2 = 0 \\ v & \text{if } b = 1 \land b_2 = 0 \\ x & \text{if } b = 0 \land b_2 = 1 \\ y & \text{if } b = 1 \land b_2 = 1 \end{cases}$$

5.3.17 V-SCT-SPEC-017: Spec: quadratic_extension — MulAddExtension

Minutes Fuzzed 120 Bugs Found 0

Natural Language We fuzzed the witness generation for the MulAddExtension() function in goldilocks/quadratic_extension.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of MulAddExtension satisfies the below formula.

MulAddExtension($a + b\sqrt{7}$, $c + d\sqrt{7}$, $e + f\sqrt{7}$) = $((ac + 7bd + e) \mod M) + ((ad + bc + f) \mod M)\sqrt{7}$.

5.3.18 V-SCT-SPEC-018: Spec: quadratic_extension — MulExtension

Minutes Fuzzed 120 Bugs Found 0

Natural Language We fuzzed the witness generation for the MulExtension() function in goldilocks/quadratic_extension.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of MulExtension satisfies the below formula.

MulExtension $(a + b\sqrt{7}, c + d\sqrt{7}) = ((ac + 7bd) \mod M) + ((ad + bc) \mod M)\sqrt{7}$.

${\bf 5.3.19\ \ V\text{-}SCT\text{-}SPEC\text{-}019: Spec: quadratic_extension} - Scalar Mul Extension$

Minutes Fuzzed	120	Bugs Found	0

Natural Language We fuzzed the witness generation for the ScalarMulExtension() function in goldilocks/quadratic_extension.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of ScalarMulExtension satisfies the below formula.

ScalarMulExtension($a + b\sqrt{7}$, c) = ($ac \mod M$) + ($bc \mod M$) $\sqrt{7}$.

5.3.20 V-SCT-SPEC-020: Spec: quadratic_extension — SubExtension

Minutes Fuzzed 120 Bugs Found 0

Natural Language We fuzzed the witness generation for the SubExtension() function in goldilocks/quadratic_extension.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of SubExtension satisfies the below formula.

SubExtension $(a + b\sqrt{7}, c + d\sqrt{7}) = ((a - c + M) \mod M) + ((b - d + M) \mod M)\sqrt{7}$.

5.3.21 V-SCT-SPEC-021: Spec: quadratic_extension — SubMulExtension

Minutes Fuzzed 120 Bugs Found 0

Natural Language We fuzzed the witness generation for the SubMulExtension() function in goldilocks/quadratic_extension.go. We tested for functional correctness by computing the expected value in a separate Go function and comparing the two results.

Formal Specification We checked the output of SubMulExtension satisfies the below formula. SubMulExtension $(a+b\sqrt{7},c+d\sqrt{7},e+f\sqrt{7})=((xe+7yf)\mod M)+((xf+ye)\mod M)\sqrt{7}$ where $x=((a-c+M)\mod M)$ and $y=(b-d+M)\mod M$.

6.1 Formal Verification Procedure

Our team verified several properties of the code. See Table 6.1 for a complete list.

We formally verified the code using Veridise's tool Picus. Picus is an open-source library that can be used to prove that zero-knowledge circuits are properly constrained* by formally verifying they have exactly one solution or find a counterexample if the circuits are not properly constrained. It implements this by a novel interaction between static analysis and SMT solvers.

Veridise auditors ran Picus on several key functions. The effort focused on the Goldilocks field emulation as most of the codebase depends on that module, and the most severe issues were identified in that region of the code (see V-SCT-VUL-001, V-SCT-VUL-002, and V-SCT-VUL-003). They also ran the tool on the quadratic extension, quadratic extension algebra, and Poseidon hash (both BN254 and Goldilocks).

To improve scalability, we modeled some constraints with a formula that can be directly reasoned with by SMT solvers. For example, we modeled several range checking constraints by replacing them with a simple inequality assertion.

Furthermore, identified under-constrained circuits (Subsections 4.1.1 to 4.1.3) could cause other circuits that include the under-constrained circuits as sub-circuits to become under-constrained as well, resulting in redundant reports. We made a fix to these under-constrained circuits first before proceeding to verify other circuits.

6.2 Properties Verified

A complete list of the properties verified is shown in Table 6.1. Each row displays a natural language description of the property proved, and its current status (i.e. verified, not verified).

Table 6.1: Formally verified properties.

ID	Property	Status	
V-SCT-FSPEC-001	Determinism of MulAdd	Verified	
	File: goldilocks/base.go	vermed	
V-SCT-FSPEC-002	Determinism of ReduceWithMaxBits	Unknown [†]	
	File: goldilocks/base.go	Ulkilowii	
V-SCT-FSPEC-003	Determinism of Exp	Verified	
	File: goldilocks/base.go	vermed	
V-SCT-FSPEC-004	Determinism of Inverse	Not verified [‡]	
	File: goldilocks/base.go		
V-SCT-FSPEC-005	Determinism of AddExtension	Verified	
	File: goldilocks/quadratic_extension.go	vermed	

^{*} In this context, *properly constrained* means not under-constrained.

Veridise Audit Report: gnark-plonky2-verifier

[†] Discovered to be not verified manually – see Subsection 4.1.3

[‡] See Subsection 4.1.1

V-SCT-FSPEC-006	Determinism of SubExtension File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-007	Determinism of MulExtension File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-008	Determinism of MulAddExtension File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-009	Determinism of SubMulExtension File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-010	Determinism of ScalarMulExtension File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-011	Determinism of InnerProductExtension File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-012	Determinism of InverseExtension File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-013	Determinism of DivExtension File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-014	Determinism of ExpExtension File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-015	Determinism of ReduceExtension File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-016	Determinism of ReduceWithPowers File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-017	Determinism of IsZero File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-018	Determinism of Lookup File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-019	Determinism of Lookup2 File: goldilocks/quadratic_extension.go	Verified
V-SCT-FSPEC-020	Determinism of MulExtensionAlgebra File: goldilocks/quadratic_extension_algebra.go	Verified
V-SCT-FSPEC-021	Determinism of PartialInterpolateExtAlgebra File: goldilocks/quadratic_extension_algebra.go Determinism of Poseidon	Verified
V-SCT-FSPEC-022	File: poseidon/bn254.go Determinism of HashNoPad	Verified
V-SCT-FSPEC-023	File: poseidon/bn254.go Determinism of Hash0rNo0p	Verified
V-SCT-FSPEC-024	File: poseidon/bn254.go Determinism of TwoToOne	Crashed [§]
V-SCT-FSPEC-025	File: poseidon/bn254.go Determinism of ToVec	Verified
V-SCT-FSPEC-026	File: poseidon/bn254.go Determinism of Poseidon	Unknown
V-SCT-FSPEC-027	File: poseidon/goldilocks.go	Unknown

[§] See Subsection 4.1.29



Fast Reed-Solomon IOPPs A transparent, succinct argument to prove [1, 2] a committed function is near a polynomial of low-degree. This is frequently used to produce a polynomial commitment scheme. See https://vitalik.ca/general/2017/11/22/starks_part_2.html for more details . 5, 83

FRI Fast Reed-Solomon IOPPs. 83

gnark A framework for zero-knowledge circuits. See https://docs.gnark.consensys.net to learn more . 1

Go An open-source language supported by Google popular in the blockchain ecosystem. See https://go.dev to learn more. 1, 6

Goldilocks Field The 64-bit field used in plonky2. 2, 5

PLONK An arithmetization strategy for zero-knowledge circuits developed in [3]. See [4] or https://vitalik.ca/general/2019/09/22/plonk.html for more details. 5, 83

PLONKish A catch-all term for arithmetization strategies for zero-knowledge circuits based off of PLONK, but slightly generalized (typically to handle more gates or a different polynomial commitment). For more details, see https://docs.zkproof.org/pages/standards/accepted-workshop3/proposal-turbo_plonk.pdf. 83

plonky2 A PLONKish arithmetization developed by Polygon Zero with various optimizations and FRI as the polynomial commitment scheme. See https://github.com/0xPolygonZero/ plonky2/ for more details . 1, 83

Satisfiability Modulo Theories The problem of determining whether a certain mathematical statement has any solutions. SMT solvers attempt to do this automatically. See https://en.wikipedia.org/wiki/Satisfiability_modulo_theories to learn more . 83

smart contract A self-executing contract with the terms directly written into code. Hosted on a blockchain, it automatically enforces and executes the terms of an agreement between buyer and seller. Smart contracts are transparent, tamper-proof, and eliminate the need for intermediaries, making transactions more efficient and secure.. 1, 83

SMT Satisfiability Modulo Theories. 81

Solidity The standard high-level language used to develop smart contracts on the Ethereum blockchain. See https://docs.soliditylang.org/en/v0.8.19/ to learn more. 1

zero-knowledge circuit A cryptographic construct that allows a prover to demonstrate to a verifier that a certain statement is true, without revealing any specific information about the statement itself. See https://en.wikipedia.org/wiki/Zero-knowledge_proof for more . 5, 83



- [1] Eli Ben-Sasson et al. 'Fast Reed-Solomon Interactive Oracle Proofs of Proximity'. In: *Electron. Colloquium Comput. Complex.* 2017 (cited on page 83).
- [2] Eli Ben-Sasson et al. DEEP-FRI: Sampling Outside the Box Improves Soundness. Cryptology ePrint Archive, Paper 2019/336. https://eprint.iacr.org/2019/336. 2019. URL: https://eprint.iacr.org/2019/336 (cited on page 83).
- [3] Ariel Gabizon, Zachary J. Williamson, and Oana-Madalina Ciobotaru. 'PLONK: Permutations over Lagrange-bases for Oecumenical Noninteractive arguments of Knowledge'. In: *IACR Cryptol. ePrint Arch.* 2019 (2019), p. 953 (cited on page 83).
- [4] Justin Thaler. 'Proofs, Arguments, and Zero-Knowledge'. In: Foundations and Trends® in Privacy and Security 4.2–4 (2022), pp. 117–660. DOI: 10.1561/3300000030 (cited on page 83).